Lesson 39: Improper Fractions and Mixed Numbers

You have been adding, subtracting, multiplying and dividing proper fractions. We can do the same operations with an improper fraction. An **improper fraction** is a fraction where the numerator value (top number) is larger than the denominator (bottom number).

Examples are: \[ \frac{5 \ 8 \ 3}{2 \ 7 \ 2} \]

You are able to convert improper fractions back to the proper form by dividing the denominator into the numerator.

Example: \[ \frac{5}{2} = 2 \frac{5}{4} \]

Answer: \[ \frac{5}{2} = 2 \frac{1}{2} \]

Now you are working with a mixed number \(2 \frac{1}{2}\).

A **mixed number** (sometimes referred to as a mixed fraction) has both a whole number and a fraction.

This illustration shows you five halves:
Now it is time for you to try converting improper fractions to mixed numbers.

**Take Lesson 39 Quiz 1**

Now you can take a mixed fraction and convert it to an improper fraction. Remember our discussion where....

\[
\frac{5}{2} = \frac{1}{2} \quad \frac{1}{2} \quad \frac{1}{2} \\
\frac{1}{2} \quad \frac{1}{2}
\]

1 whole + 1 whole + \(\frac{1}{2}\) = \(2 \frac{1}{2}\)

To convert a mixed number, you need to break it down to see how many parts you can make from the mixed number. In \(2 \frac{1}{2}\), you are working with halves. 2 wholes = how many halves?

\[
\frac{1}{2} \quad \frac{1}{2} \\
\frac{1}{2} \quad \frac{1}{2}
\] = \(\frac{4}{2}\) halves
Then add the fraction \[\frac{1}{2}\] to \[\frac{5}{2}\]

There is a trick to learn to do this step easily. In the mixed number \(2\frac{1}{2}\), multiply the whole number by the denominator, then add the numerator.

Example:

Step 1) Multiply 2 (whole number) \(\times\) 2 (denominator) = 4

Step 2) Add + 1 (mixed number numerator) + 1

You now have the numerator of the improper fraction \(\rightarrow\) 5

Step 3) Bring the denominator over, making it \(\frac{5}{2}\)

Let’s change some mixed numbers to improper fractions!

Example 1:
\[
\frac{3}{2} \cdot \frac{7}{2} = \frac{7x2}{2} = 6 + 1 = 7
\]

Example 2:
\[
\frac{7}{8} \cdot \frac{59}{8} = \frac{7x8}{8} = 56 + 3 = 59
\]

Now, let’s change the improper fraction back to a mixed number!

Example 1:
\[
\frac{7}{2} \rightarrow 2\frac{3}{6} \rightarrow 3\frac{1}{2}
\]

Example 2:
\[
\frac{59}{8} \rightarrow 8\frac{1}{8} \rightarrow 7\frac{3}{8}
\]

Take Lesson 39 Quiz 2
Add:

\[ 2 \frac{5}{8} + \frac{4}{8} \]

\[ = 2 \frac{9}{8} \]

Since \( \frac{9}{8} \) is improper, we can convert by dividing the denominator into the numerator and get a mixed number. Then, add to the whole number.

\[ \frac{1}{8} \]

\[ \overline{9} = 1 \frac{1}{8} \]

Answer:

\[ \frac{2}{8} + 1 \frac{1}{8} \]

\[ = 2 \frac{9}{8} = 1 \frac{1}{8} \]

\[ + 2 \]

\[ = 3 \frac{1}{8} \]

This is your final answer in correct form.

Let's try multiplication!

Multiply:

\[ 2 \frac{1}{2} \times 1 \frac{1}{8} \]

Both are mixed numbers. You need to convert both to improper fractions in order to carry out the operation.
Now multiply:

\[
\frac{5}{2} \times \frac{9}{8} = \frac{45}{16}
\]

Change the improper fraction above to a mixed number.

\[
\frac{2}{16} \times \frac{32}{16} = 2 \frac{13}{16}
\]

(The answer is complete. No need to reduce the fraction.)

Now you try!

**Take Lesson 39 Quiz 3**

Try this – Divide mixed numbers!

Did you remember all the steps?

Step 1 - convert mixed numbers

Step 2 – invert (flip) the second fraction and multiply

Step 3 - convert the improper fraction back to a mixed number in lowest terms

\[
1 \frac{1}{2} \div 1 \frac{1}{6} \quad 3 \div 7 \quad 3 \times 6 = 18 \quad 1 \frac{4}{14} \quad 1 \frac{2}{7}
\]

Great! Now you try to solve some problems.

**Take Lesson 39 Quiz 4**